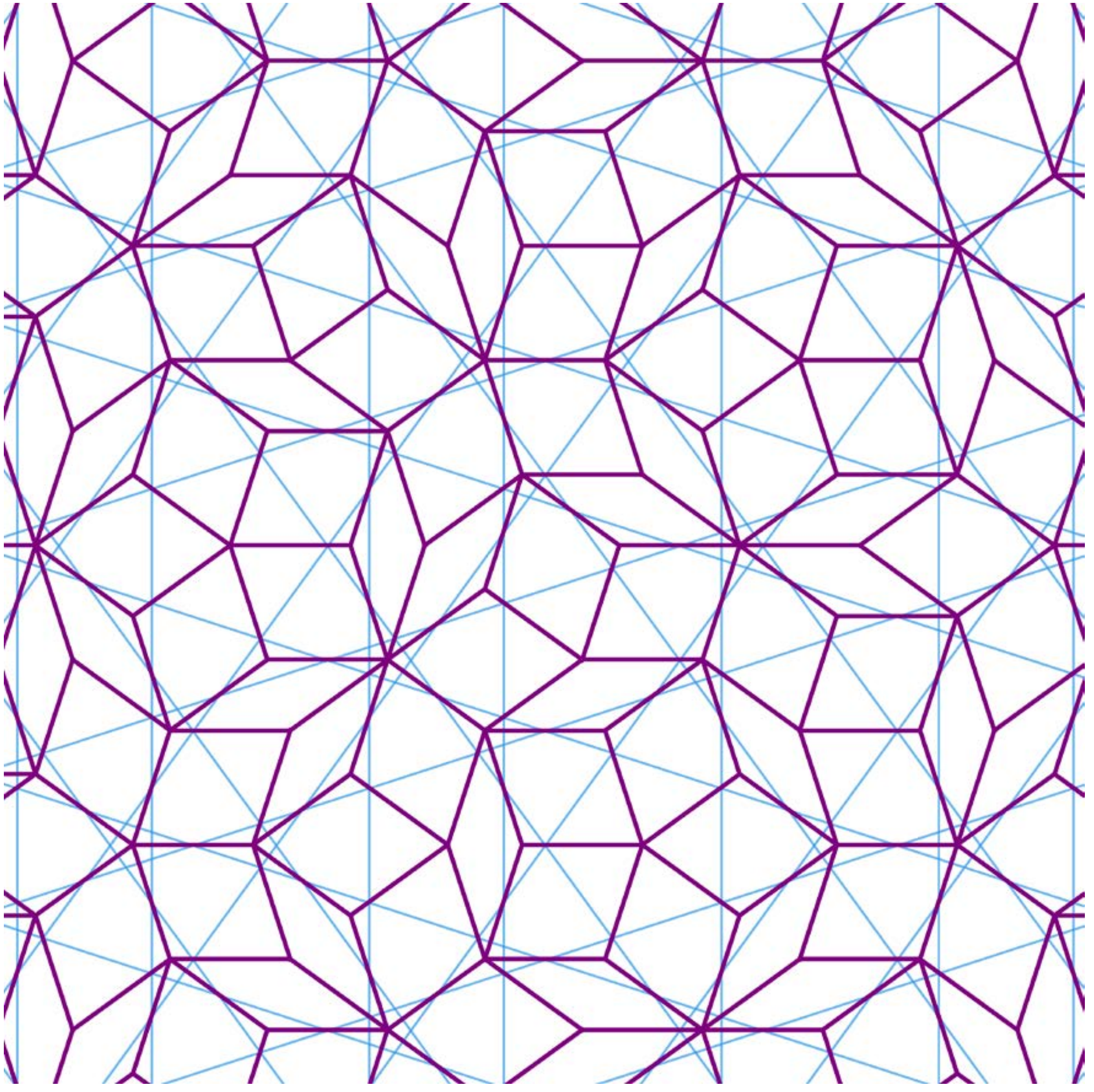
Ammann-Penrose tilings and physics: new connections and open problems

Latham Boyle (Perimeter Institute)

(with Madeline Dickens, Paul Steinhardt, Felix Flicker)

The Ammann-Penrose Tiling



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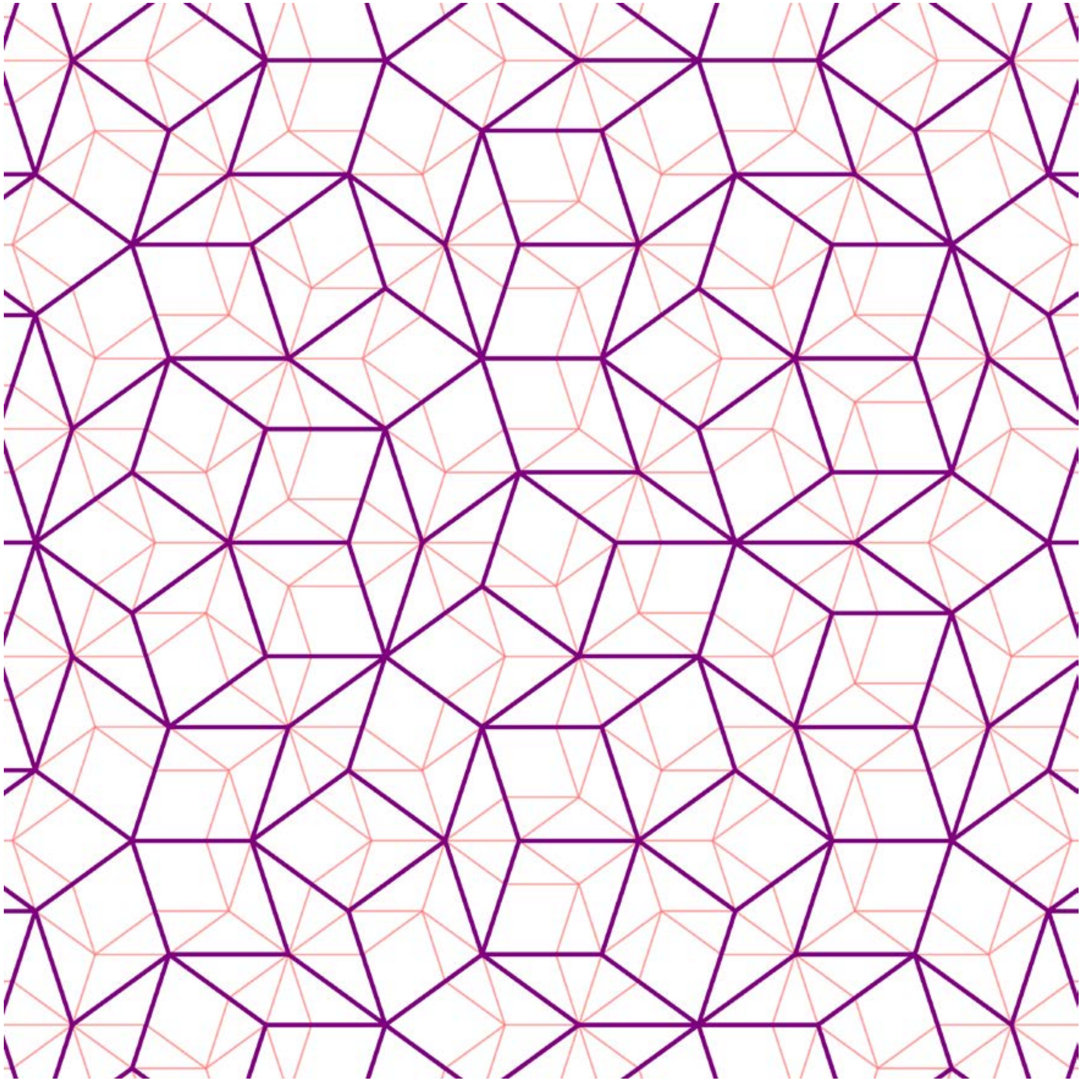

See: “The Mysterious Mr. Ammann” by M. Senechal

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An Unorthodox Explanation of the Cretaceous-Tertiary Boundary Event

No known fossil from the Cretaceous era had sufficiently large brain size to organize governments and invent nuclear weapons. However, there are possible lines of evolution that could reasonably have led to such a creature along a path resembling the path of human evolution. The Jurassic Archaeopteryx had opposable digits on the front limbs, which have been described as useful for tree-climbing (1), and had the largest brain to body ratio of any known creature of the period (2). Its recently discovered Cretaceous descendants, the Enantiornithes (3), had front limbs so large they were obviously used in locomotion, are believed to have had no flying ability, had a humerus greatly resembling that of a modern primate, (showing probable similar use of the shoulder and elbow joints), may reasonably be assumed to have kept the ancestral opposable digits, had a metatarsal which shows signs of having been a base for two roughly parallel toes and a third toe extending inward at an angle of about 60 degrees, had a "wingspan" of around 1 meter, about the size of an average monkey, and had a geographical distribution similar to modern monkeys (unknown in the American West or Mongolia, the dominant form of life in forests 27 degrees from the Equator).

The brain size is unknown. However, since their ancestor over 75 million years prior to the end of the Cretaceous had a brain size larger than that of 65-million year old mammals (2,4), and since modern angiosperms with fruit and edible leaves had appeared 35 million years before the end of the Cretaceous (5), creating a treetop environment similar to that which caused the evolution of higher primates, there is no reason why similar evolution would not have occurred.

Since the treetops of the late Cretaceous provided as much food as the modern treetops, and where there is food animals will evolve to eat it, if the Enantiornithes were not monkey-like treetop dwellers there was probably another such group which has not been found. The fauna of the equatorial regions of the late Cretaceous is little known.

Early hominids were extremely rare, and confined to the continent of

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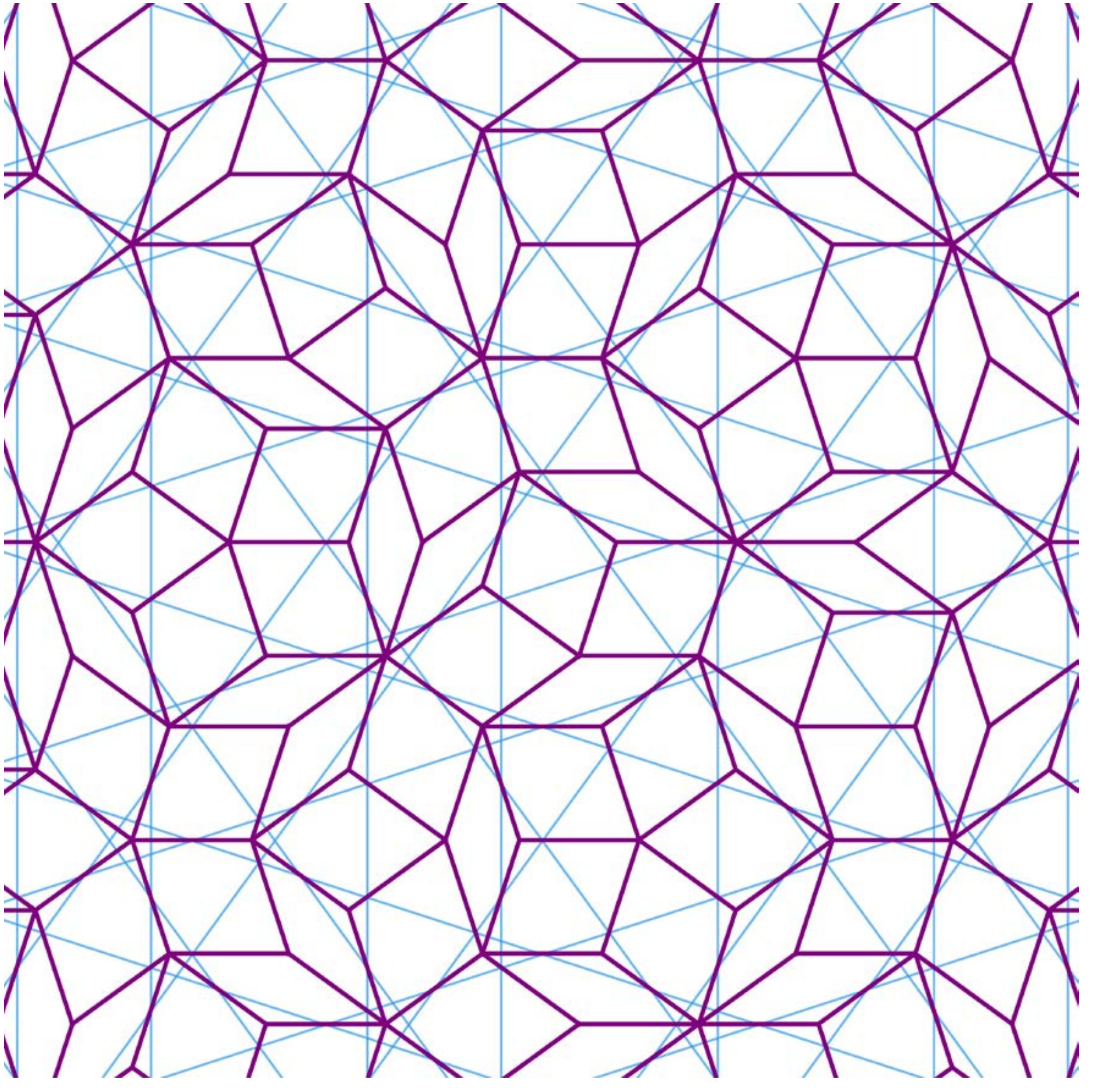
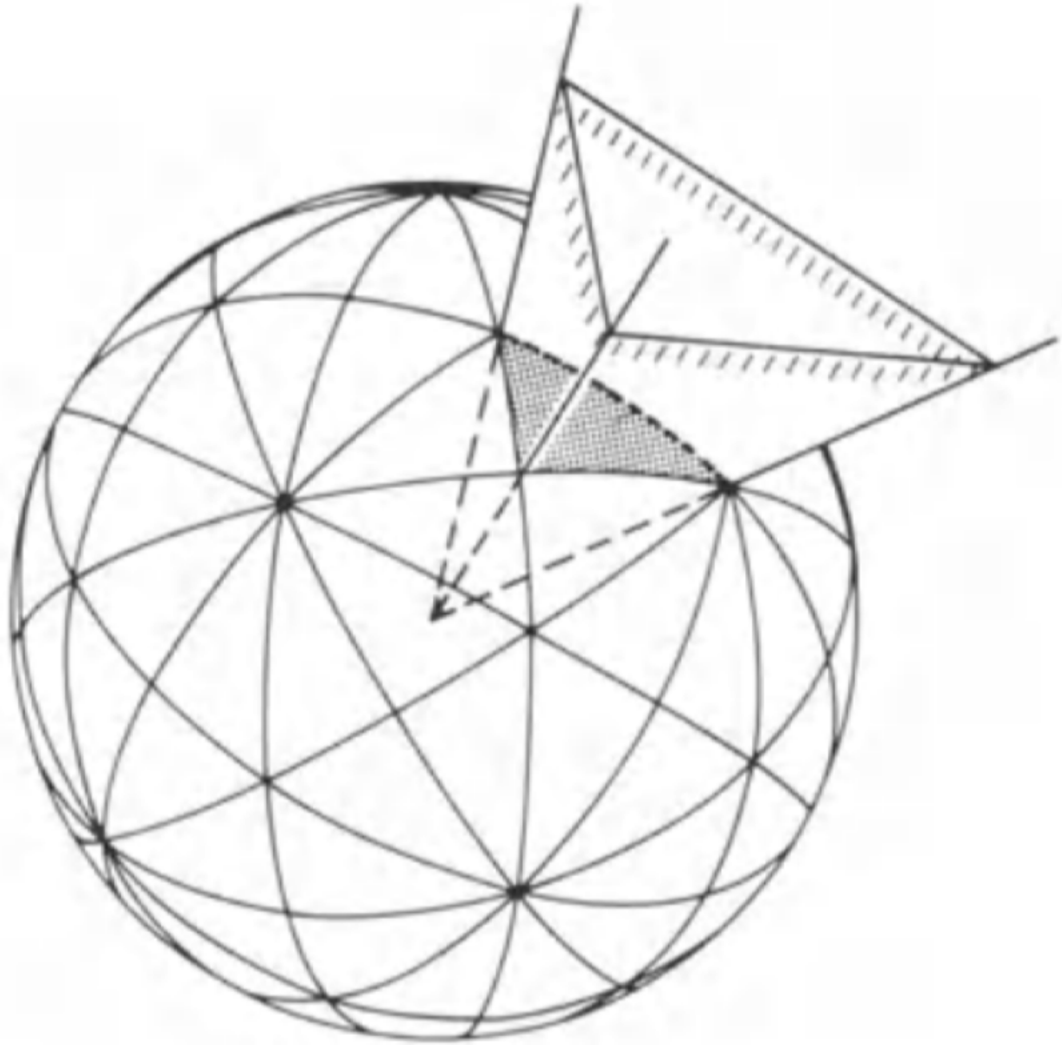
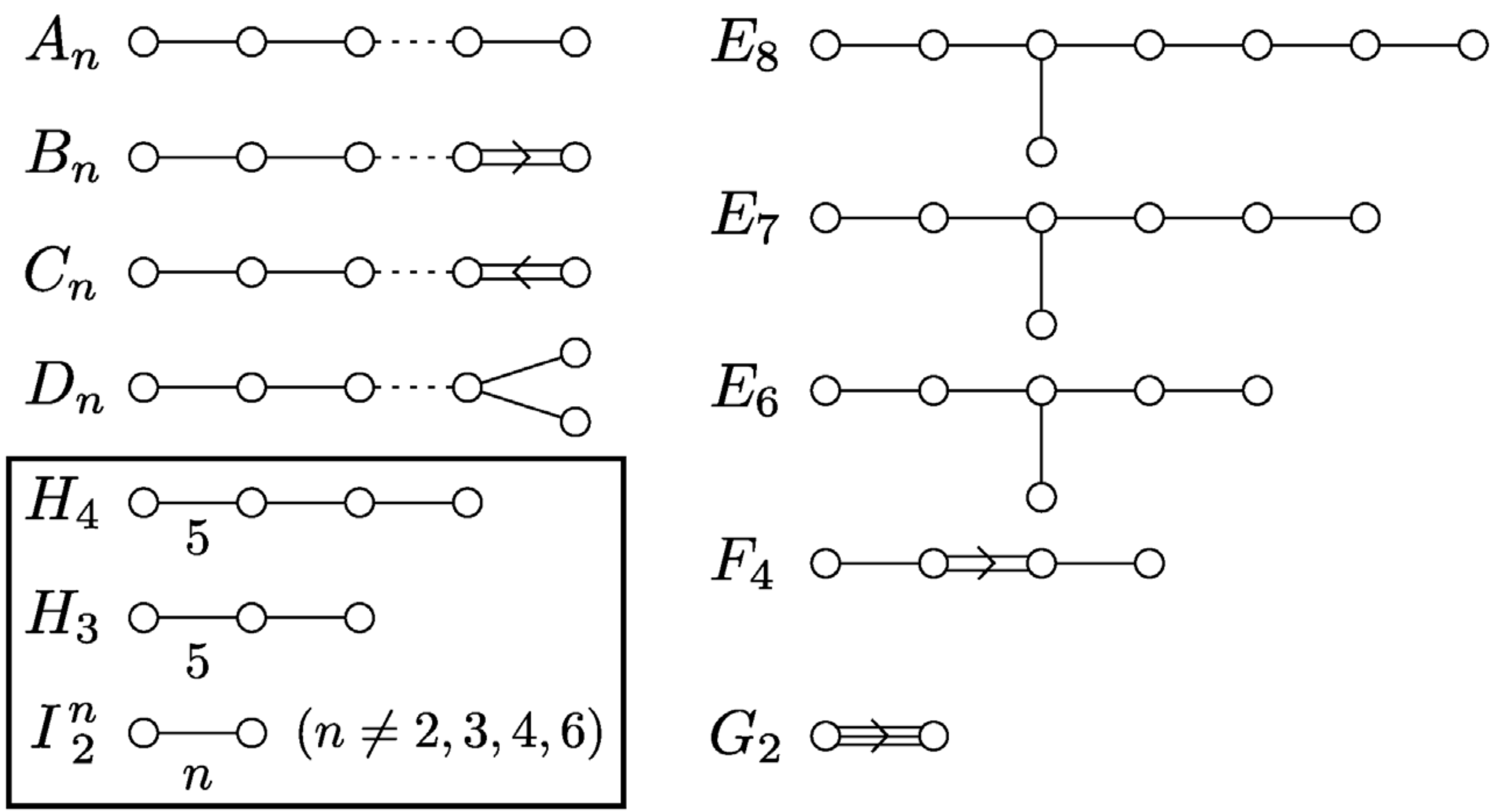


Reflection Groups and Root Systems

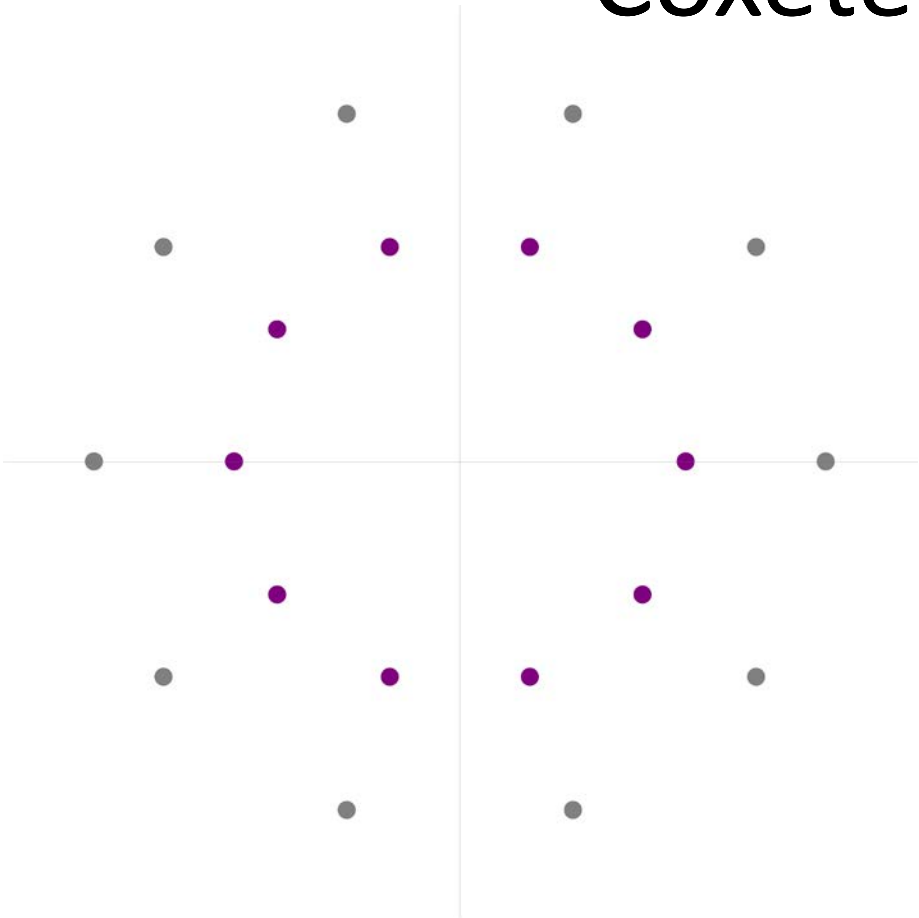


(see Conway&Sloane)

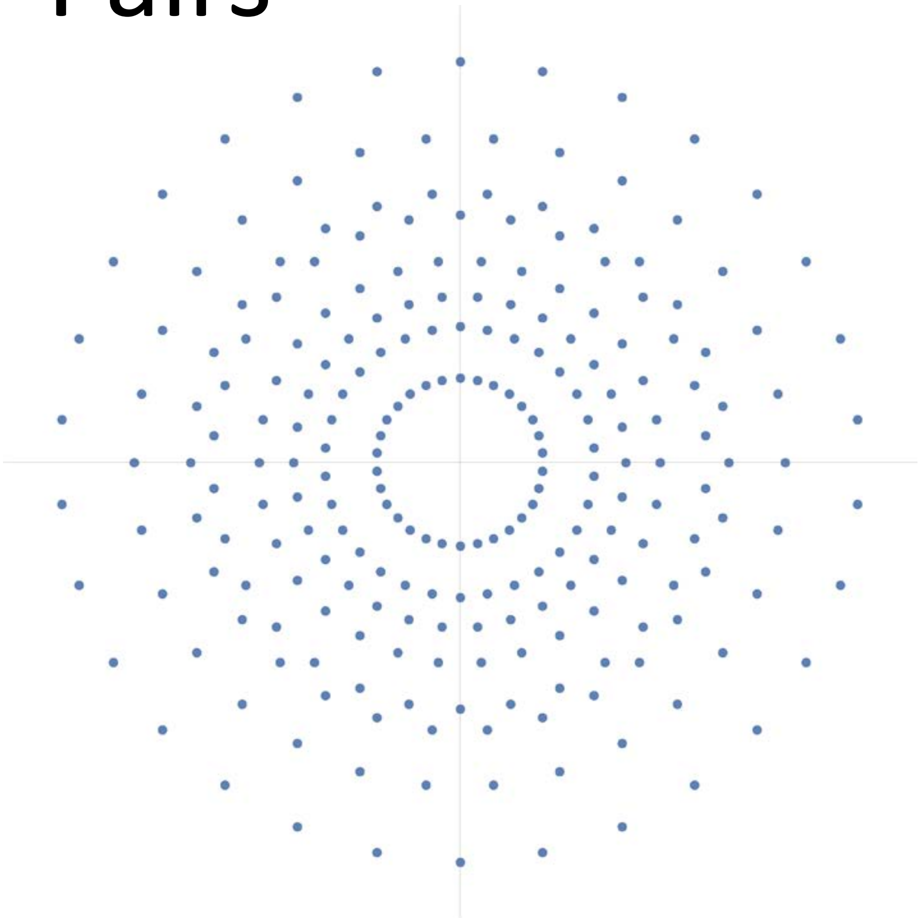
Coxeter-Dynkin Diagrams:



Coxeter Pairs

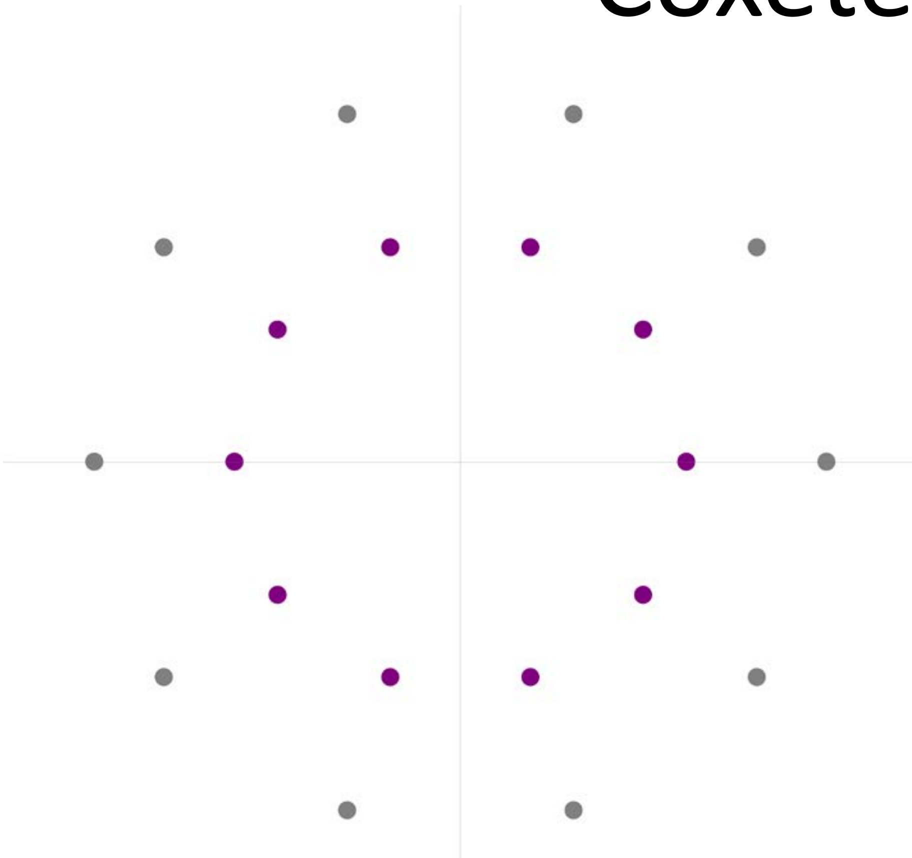


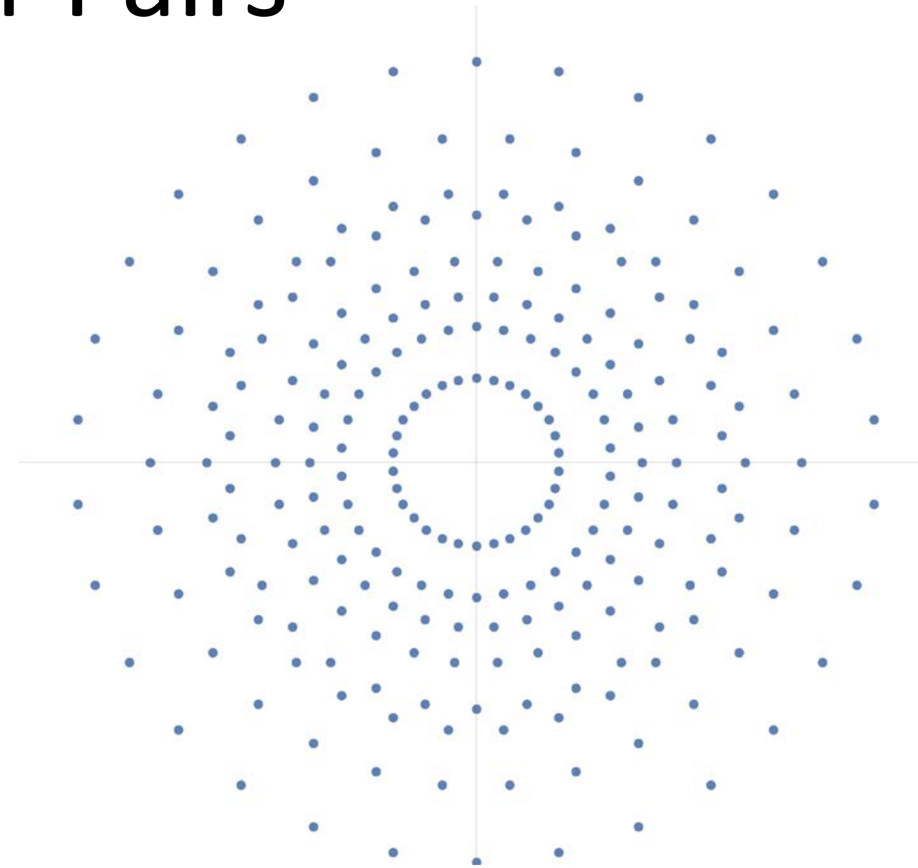
$$A_4 \leftrightarrow I_2^5$$



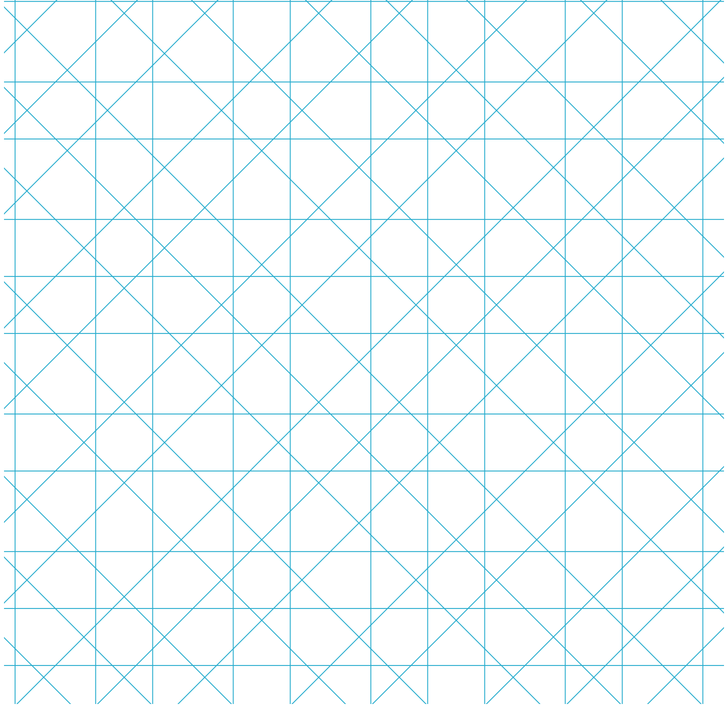
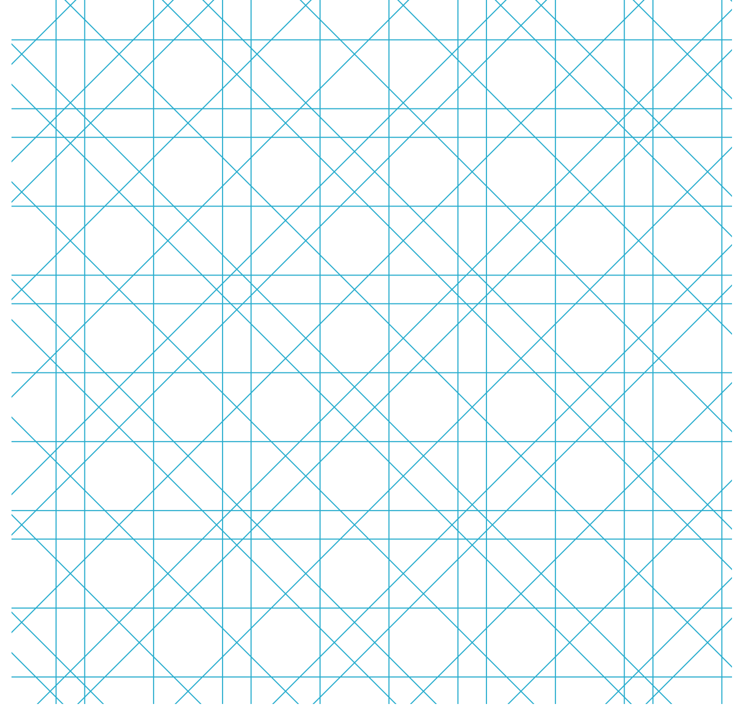
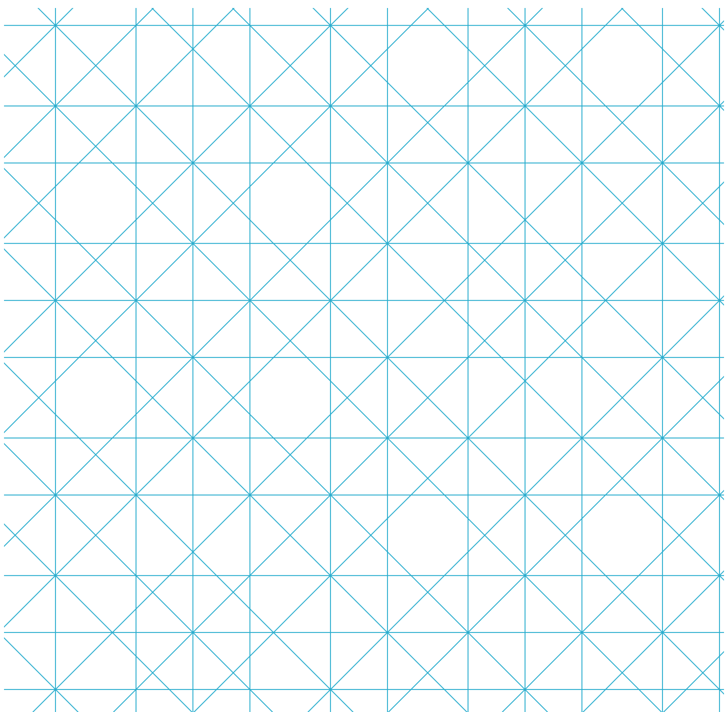
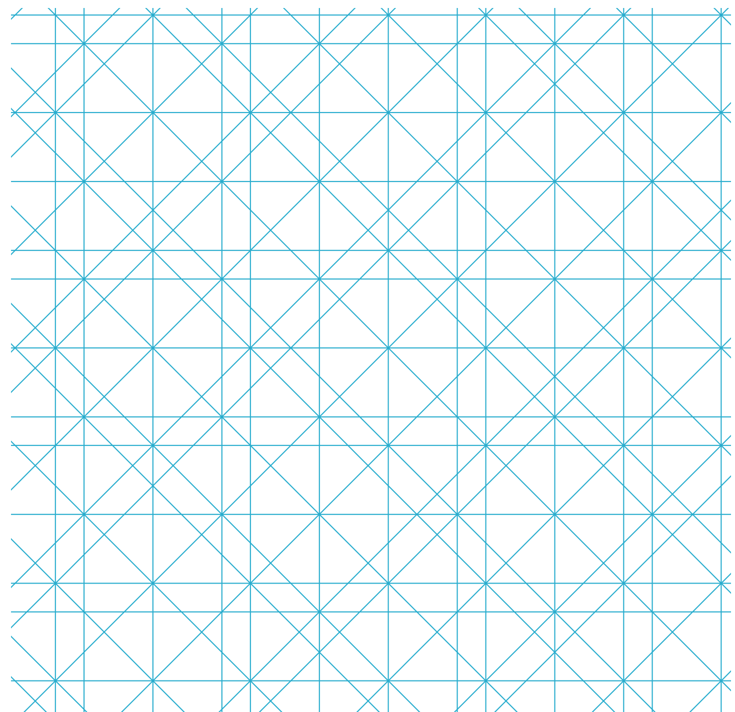
$$E_8 \leftrightarrow I_2^{30}$$

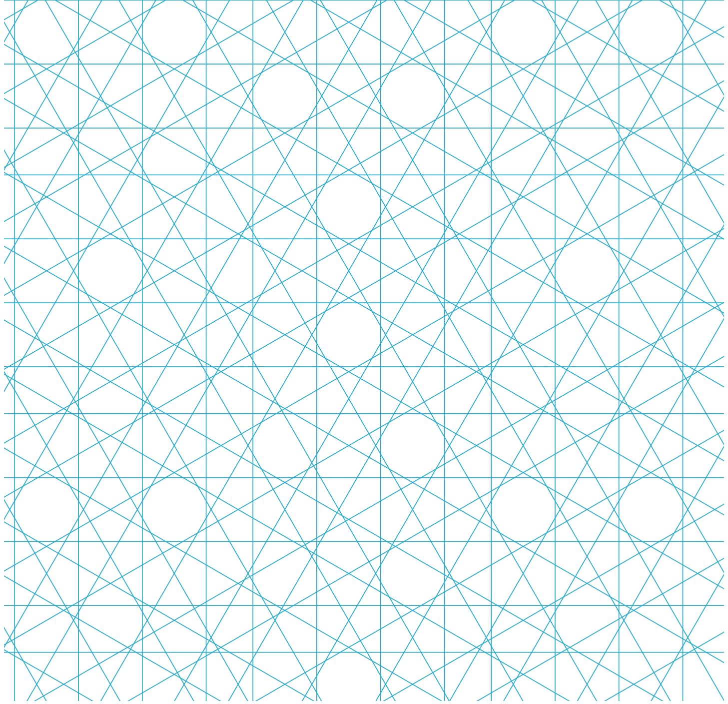
Coxeter Pairs

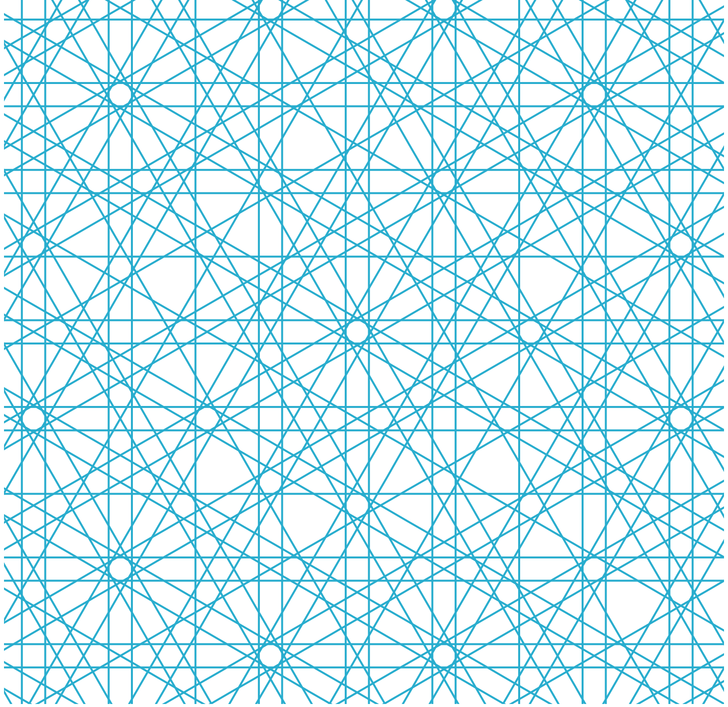


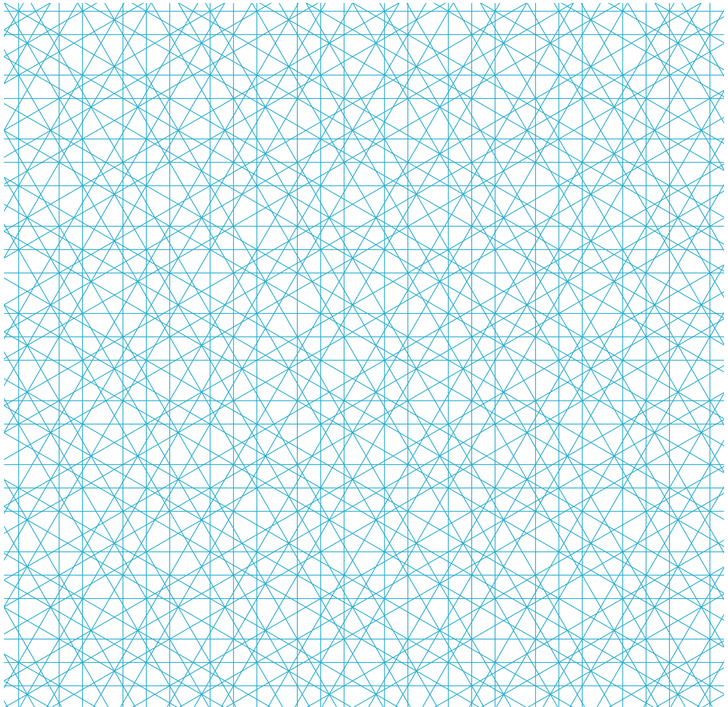


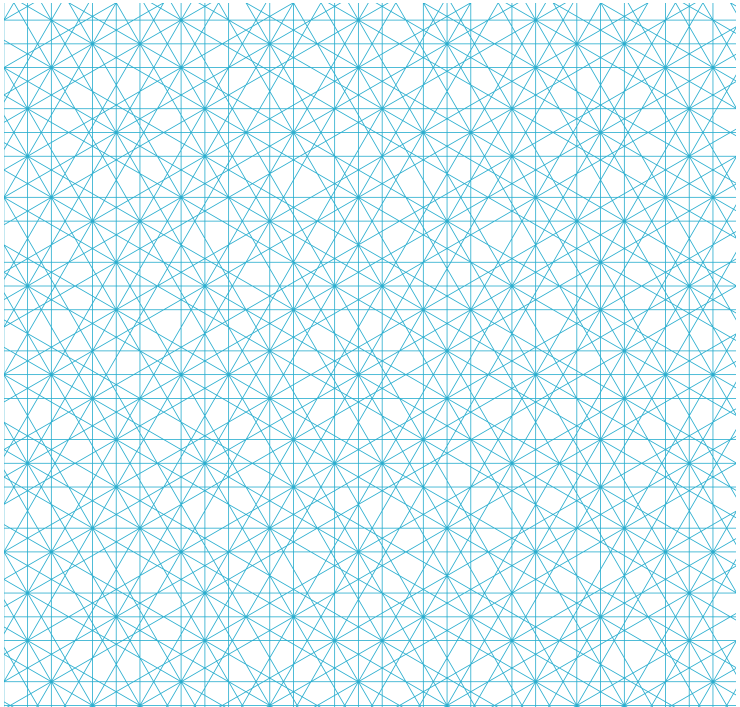
non-crystallographic root system $\theta^{\parallel}$	crystallographic partner θ	degree $N = d/d^{\parallel}$
I_2^p (p any prime ≥ 5)	A_{p-1}	$(p-1)/2$
$I_2^{2^m}$ (m any integer ≥ 3)	$B_{2^{m-1}}/C_{2^{m-1}}$	2^{m-2}
I_2^{12}	F_4	2
I_2^{30}	E_8	4
H_3	D_6	2
H_4	E_8	2

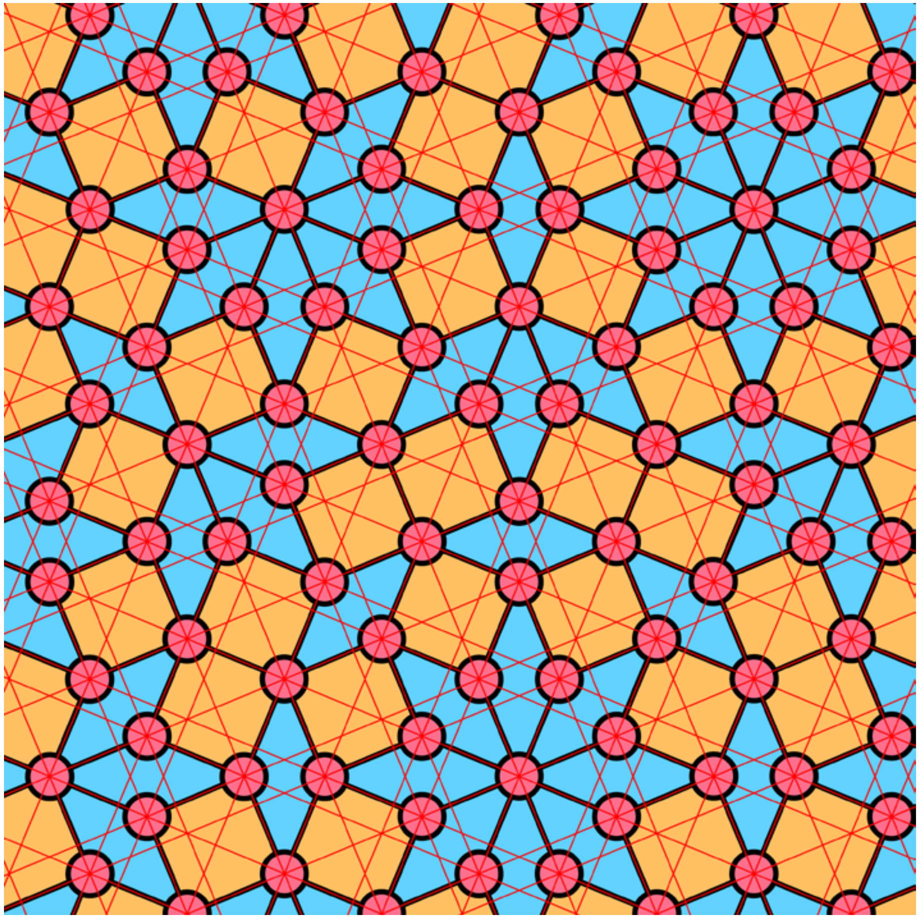





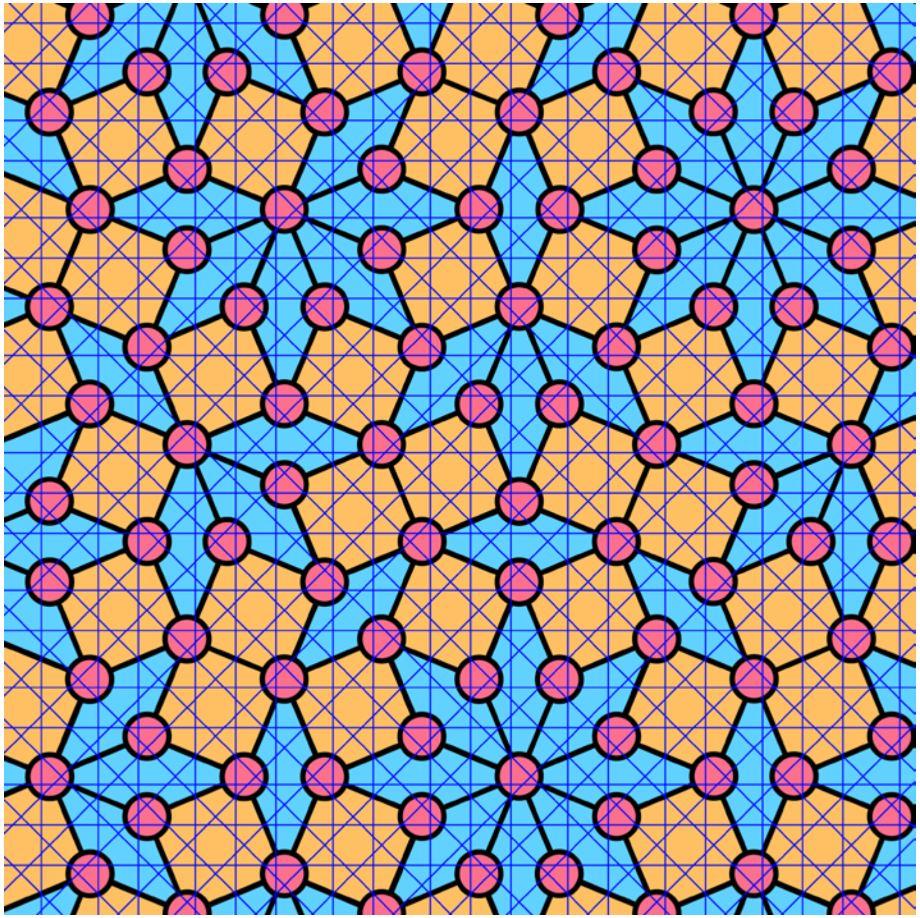












Applications?

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Coxeter Pairs Ammann Patterns and Penrose Like Ti Pattern Spandex Trolley Case Travel Luggage Protector Suitcase...

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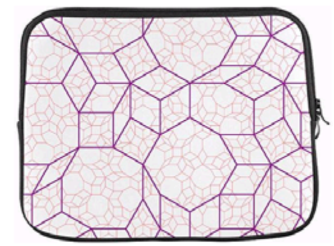

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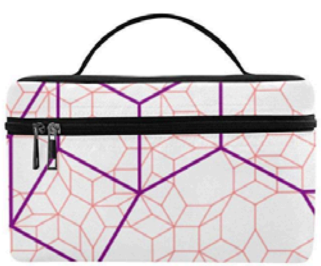

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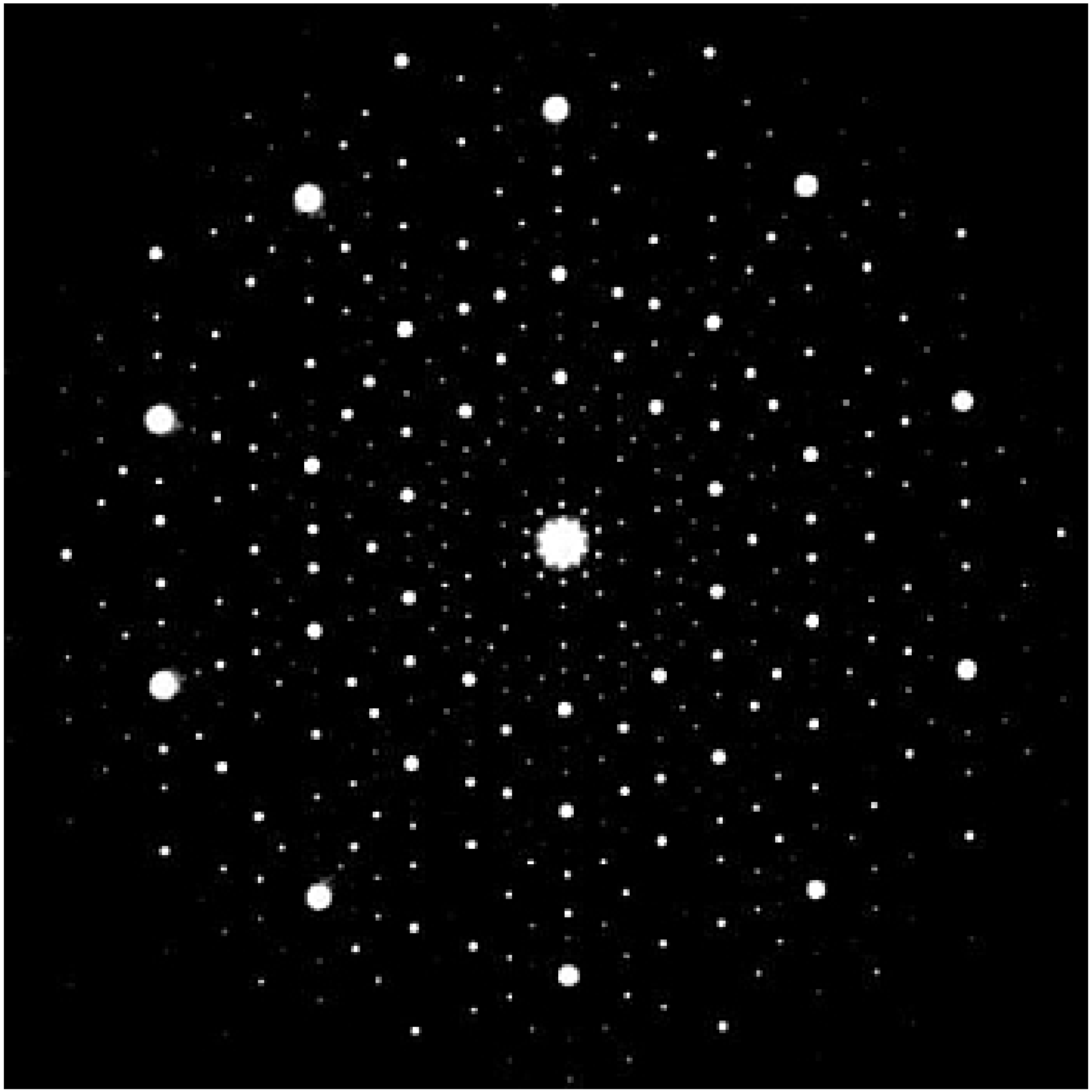
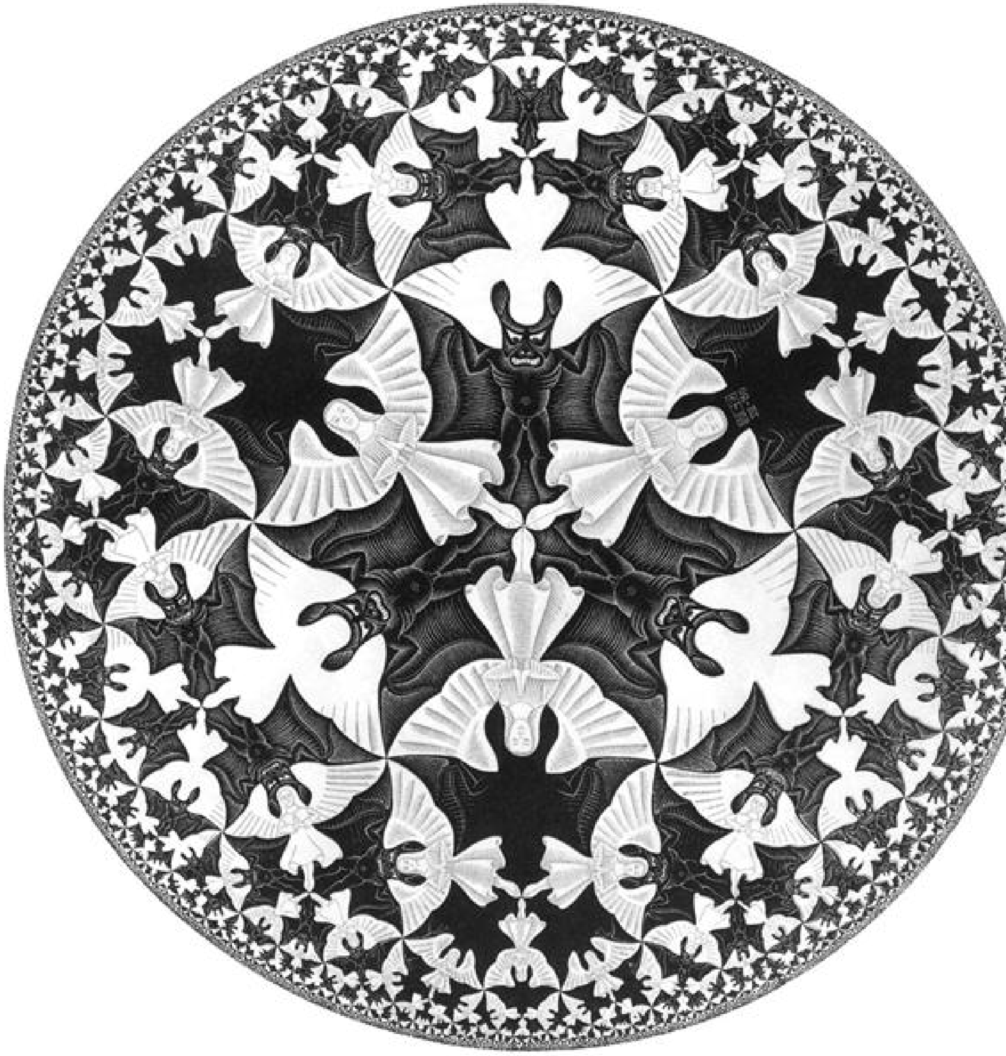
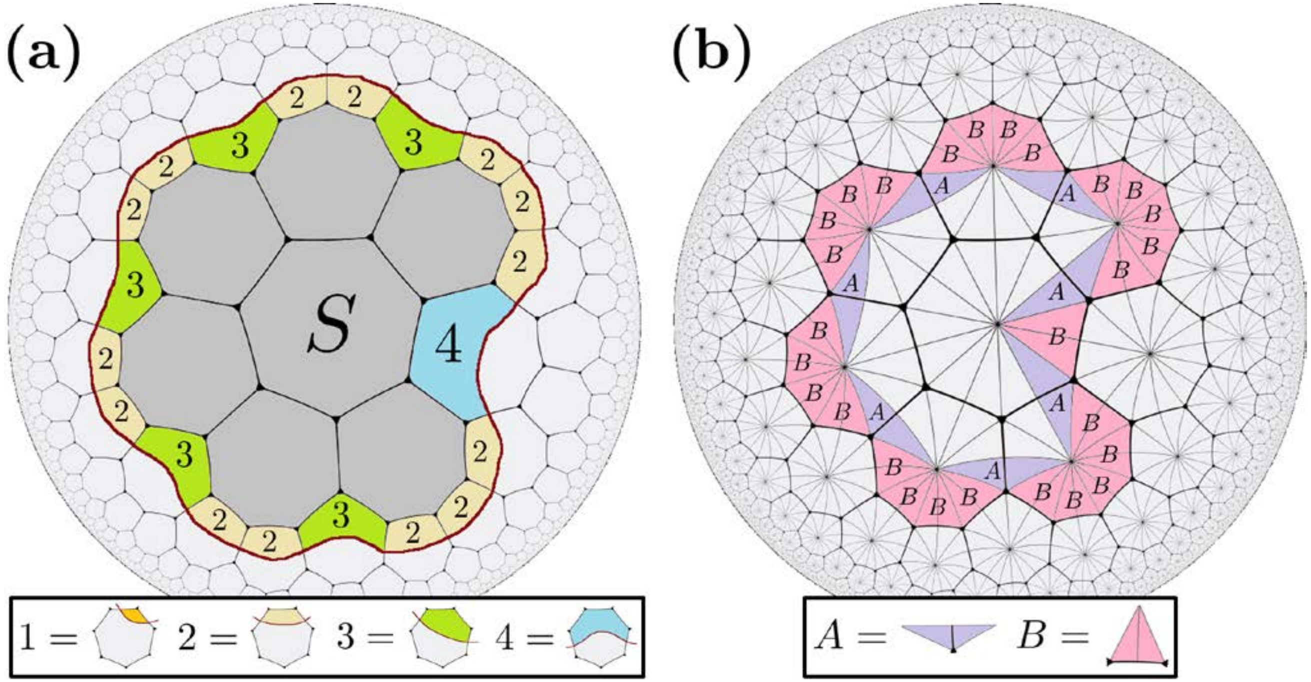
Nobel Prize 2011 (D. Schectman)







(c)

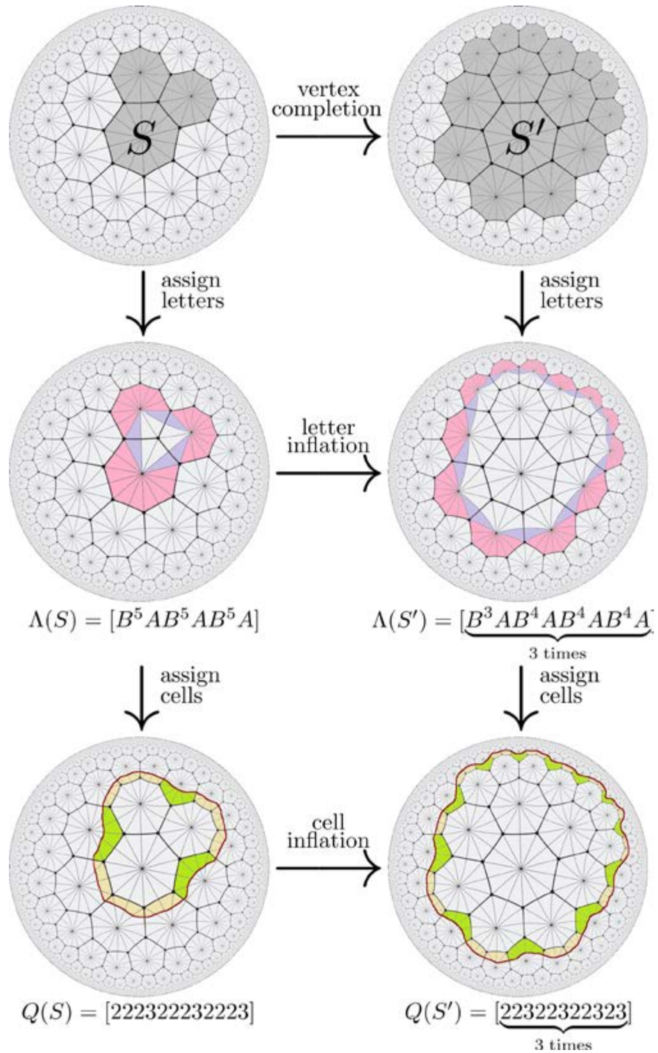
$Q(S) = [42223223223223223222]$

$\Lambda(S) = [BABABBBBABBBBABBBBABBBBABBBBABBBB]$

(d)

$2 \leftrightarrow B \quad 3 \leftrightarrow BAB \quad 4 \leftrightarrow BABAB \quad \cdots \quad n \leftrightarrow B\,\underbrace{AB \ldots AB}_{n-2 \text{ times}}$

(arXiv:1805.02665, LB, M. Dickens, F. Flicker



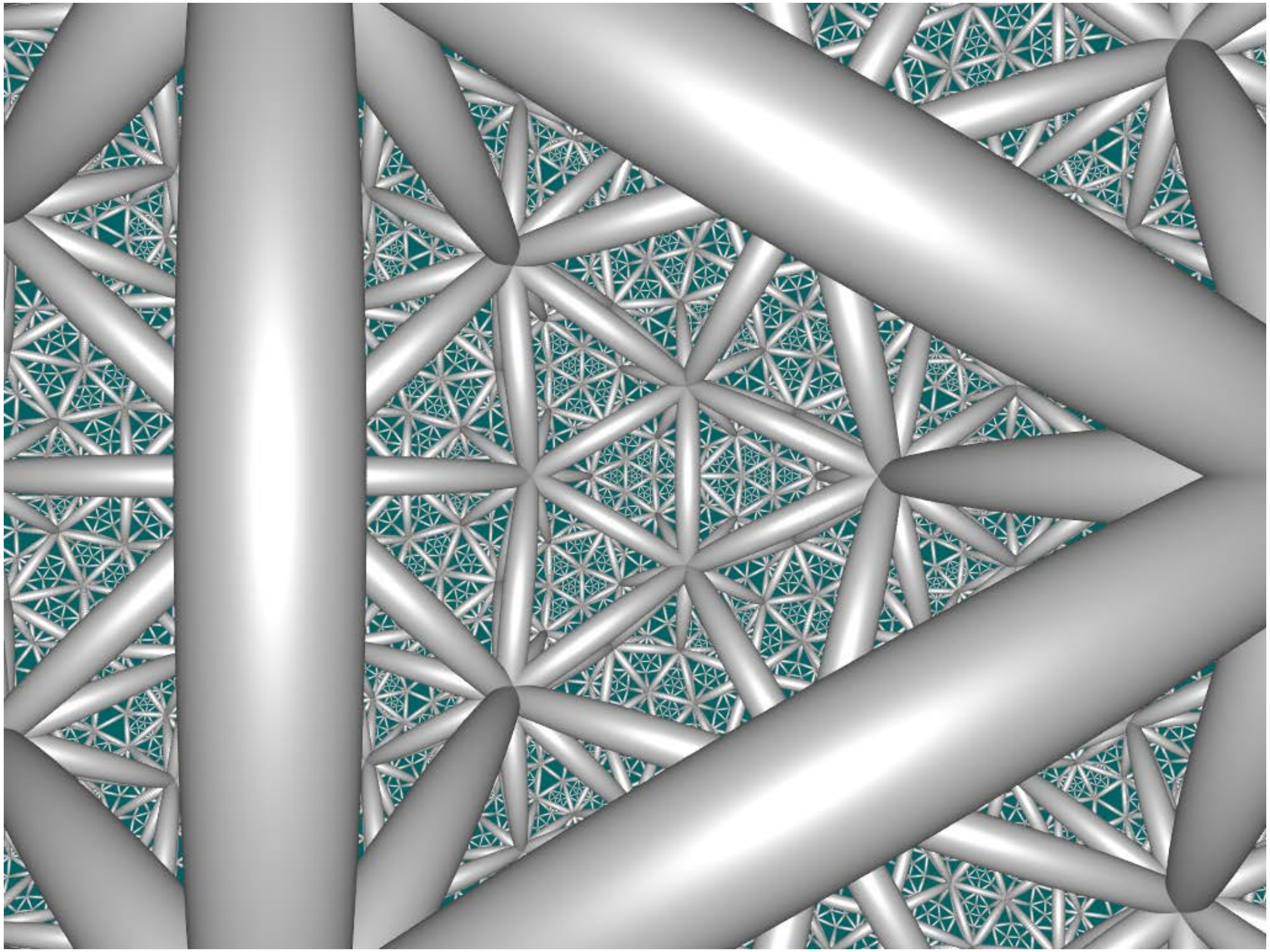
$$A \to A^{-1}B^{-5}$$

$$B \to B^4 A$$

$$2 \to 223$$

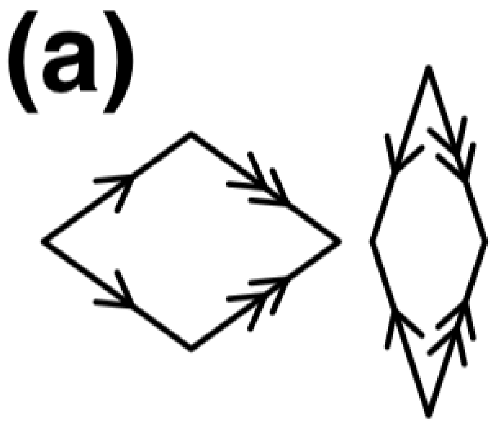
$$3 \to 23$$

Problem #1: higher dimensions



(see Coxeter: “Regular Honeycombs in Hyperbolic Space”)

Problem #2: ”Decapod Defects”



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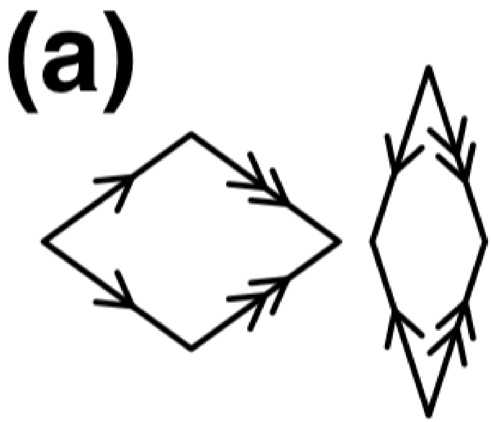

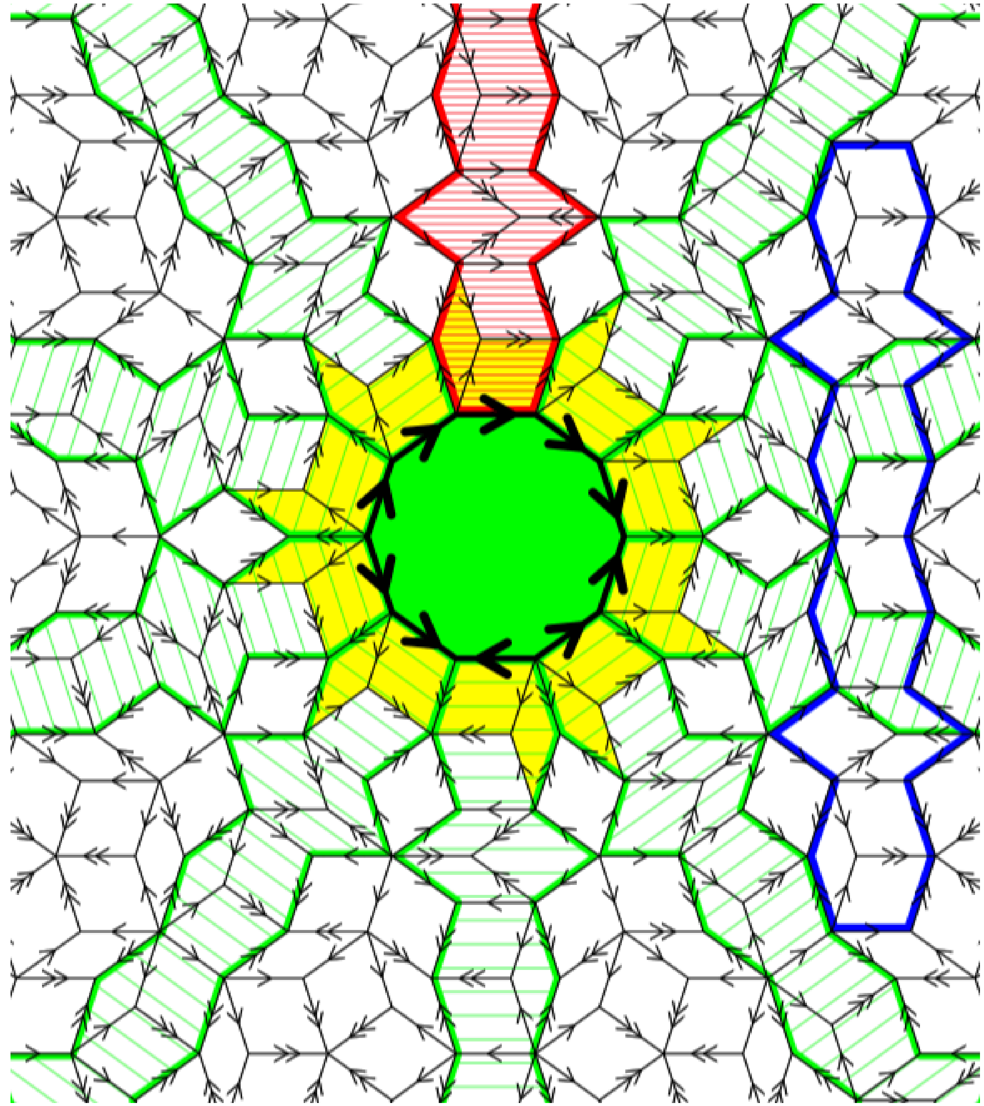

Nonabelian anyons?

Problem #3: Quasicrystalline Compactifications

QUASICRYSTALLINE COMPACTIFICATION

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Received 14 October 1987

A class of asymmetric orbifolds is constructed using ideas from the theory of quasicrystals. These orbifolds provide examples of solutions to string theory which are isolated in the sense that they do not belong to a continuous moduli space of solutions and moreover cannot be approximated by rational orbifolds. One of the notable features of the construction is that many aspects of the models are easily handled with the theory of cyclotomic fields.

Problem #3: Quasicrystalline Compactifications

$$X = X^i_L + X^i_R \qquad i = 1..D$$

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$$X^i_L = x^i_L + p^i_L(t+\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{\tilde{\alpha}^i_n}{n}e^{-2in(t+\sigma)}$$

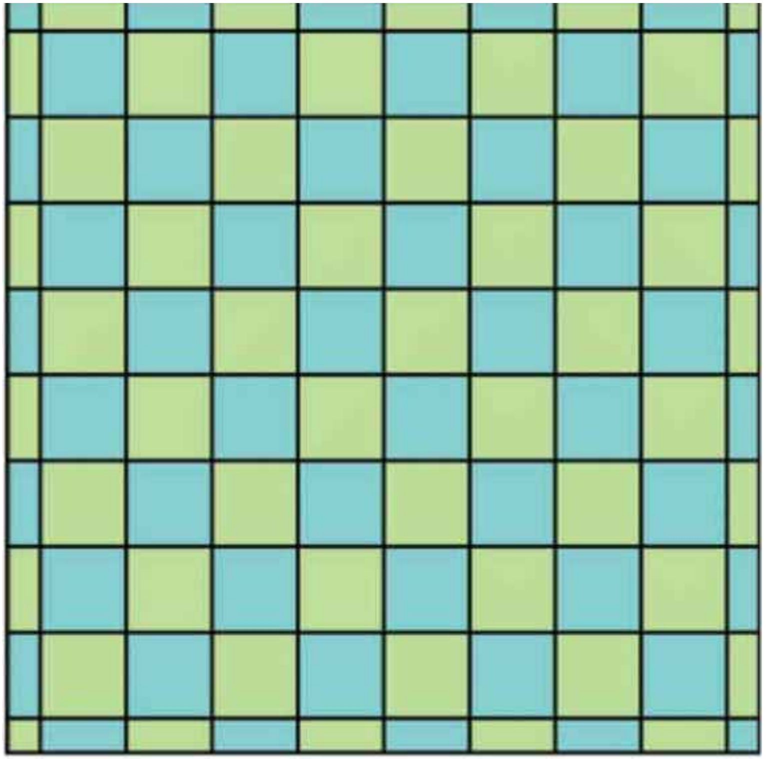
$$X^i_R = x^i_R + p^i_R(t-\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{\alpha^i_n}{n}e^{-2in(t+\sigma)}$$

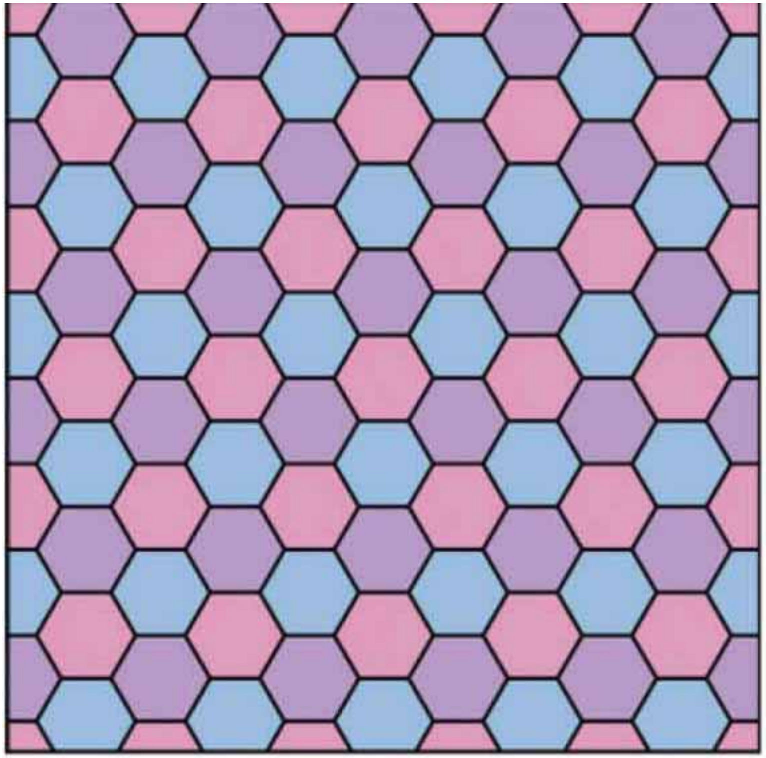
$$(p_L, p_R) = (p/2 + w, p/2 - w)$$

Summary

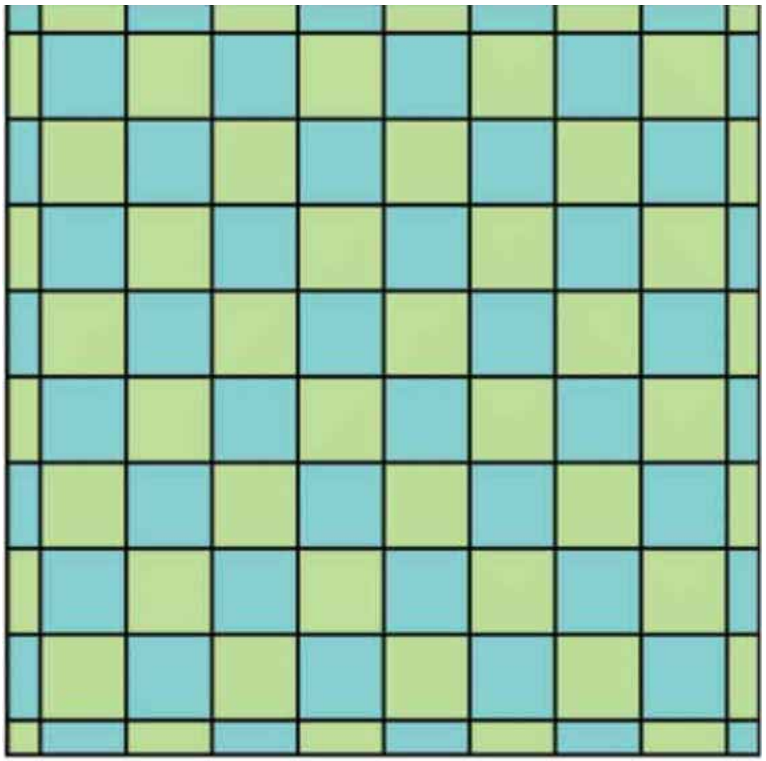
- Ammann-Penrose Tilings (from Coxeter Pairs)
- Problem 1: At the boundary of hyperbolic space?
- Problem 2: (Nonabelian) anyons? (quantum computing)
- Problem 3: In string compactifications?

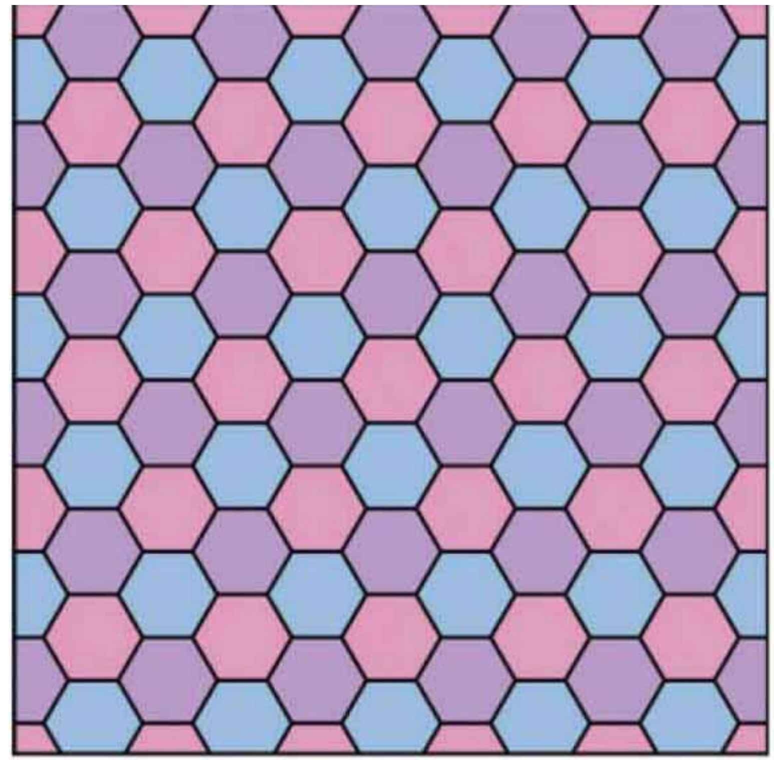
Tilings with translational symmetry:





Tilings with translational symmetry:





Theorem: only 2, 3, 4 or 6-fold symmetry is allowed!

Fibonacci quasilattices: